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Parametric Sensitivity of MEMS-gyro

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Abstract. The paper presents the finite element (FE) modeling approach to sensitivity analysis of MEMS-based gyros. The FE model is employed to both studying the system's dynamical properties and appreciation of the sensitivity of these to various influencing effects. The sensitivity functions have been obtained for adjusting the geometric parameters of the piezoelectric transducers in order to achieve the desired values of natural frequencies. Results are presented in terms of sensor performance characteristics for various design parameters and modes of operation. The modeling assumptions adopted are tested experimentally on a cantilever-shape test vehicle.

Introduction

MEMS is an enabling and extremely diverse technology that may potentially offer improved system performance and reliability, and allows the development of the next generation smart devices or systems. The small size and weight along with reduced power consumption as well as cost benefits associated with MEMS are the key features that have prompted interest primarily in the automotive industry, telecommunications sector and medical markets.

Some specific applications of the MEMS devices, e.g. motion detection ones, encounter, however, unique challenges due to both the demanding operational and environmental standards and stringent reliability and performance degradation requirements [1]. Reliability considerations are of immense importance in widening implementation of such systems.

The genuine status of existing MEMS sensors/actuators has to be defined by addressing critical questions related to application challenges, including performance determination and metrological assessment under realistic operating conditions. To create predictive reliability models the sensitivity of MEMS structures to internal variations and external influences needs to be determined.

The development of a testing infrastructure to perform reliability experiments on MEMS devices and investigation of failure modes in different environments are reported [2]. The overall goals of this work were a) to perform statistical reliability characterization of MEMS devices, b) to identify failure modes from a physics perspective, c) to develop reliability test structures, sensitive to certain aspects of failure, d) to develop predictive reliability models and e) to develop failure analysis techniques for MEMS.

Failure modes caused by mechanical and electrical instabilities in a wide range of surface micromachined microelectromechanical actuators were examined [3].

The issue of shock robustness in silicon microstructures has been addressed [4]. The vibration failure analysis of MEMS subjected to a vibration environment has been presented [5].

The complexity of MEMS and their high level of integration necessitate the development of new reliability assessment methodologies based on failure identification, observation, modeling and understanding. Standard tests for MEMS technologies are not available as yet. The use of modeling and simulation tools facilitates (along with testing and validation techniques) MEMS performance assessment and optimization.

A reliability assessment method that aims to evaluate the reliability of MEMS components by means of simulation has been presented [6]. The methodology is derived from the Physics of Failure approach and based on the idea that physical laws govern failure. A good knowledge of failure mechanisms gives the possibility to the designer to predict possible failure sites and modes by simulation on a system level.

The purpose of this paper is to explain the approach developed [7] in the field of MEMS in order to provide accurate simulation of dynamical behavior of the MEMS structures and appreciation their parametric sensitivity.

It presents the modeling of operation of a rotational motion sensor that uses a balanced oscillator (quartz tuning fork) to sense the angular rate. Other examples investigated in this work include a ring-shaped piezoelectric transducer able to measure angular velocity of rotation about its diameter and a small "cloverleaf" resonator structure suspended on a pair of orthogonal "springs" that are the Si beams.

Resonating Structures

MEMS resonators are usually mechanical vibrating systems with a high value of mechanical Q-factor; their performance and dynamic features depend substantially upon natural frequencies and shapes of vibration. Accurate simulation of MEMS transducers requires adequate models that deal not only with fundamental principles underlying measurement systems but also with principles of treatment of deviations and uncertainties based on the analysis of instrument structure and the effects of sensitivity to parameters variations and external influences. The development of the MEMS device requires a dedicated research effort to find a suitable solution.

The common feature of all such systems is their geometric symmetry and inherent presence of multiple vibration modes in the working frequency range. The tolerance in geometric and electromechanical parameters of microstructures may change mutual positions of resonance peaks and influence the width of them as well as cause uncertainties in the phase of the measured output signal. In this work we investigate the parametric sensitivity of resonant structures by using finite element dynamic models of MEMS-gyro. The parametric sensitivity functions are obtained as gradients of the properly posed target function providing a quantitative measure of the precision of measurement. The obtained results are verified by performing direct dynamic/harmonic analysis of the model developed at different values of parameters.

Fig. 1 illustrates the different resonating structures of the MEMS-based gyros that measure the rotation of a body with respect to a reference system by means of the Coriolis force.

Finite Element Model

The MEMS-based gyros are vibrating piezoelectric devices of a complex geometric shape. Surfaces of piezoelectric transducers are laminated by electrodes that enable one to create or to register electric fields inside of the material. The differential equations governing the behavior of the piezoelectric continuum are the Newton's laws of motion and the quasi-static approximation of the Maxwell's equation as:

$$\begin{cases}\{\boldsymbol{\sigma}\} = \left[\mathbf{c}^{E}\right]\{\boldsymbol{\varepsilon}\} - [\mathbf{e}]\{\mathbf{E}\},\\ \{\mathbf{D}\} = [\mathbf{e}]^{T}\{\boldsymbol{\varepsilon}\} - [\boldsymbol{\kappa}]\{\mathbf{E}\},\end{cases} \tag{1}$$

Where $\{\mathbf{E}\}$ is electric field vector, $\{\mathbf{D}\}$ is electric displacement vector, $\left[\mathbf{c}^{E}\right]$ is stiffness tensor at constant electric field value, $[\mathbf{e}]$ is piezoelectric tensor, $[\boldsymbol{\kappa}]$ is dielectric tensor.

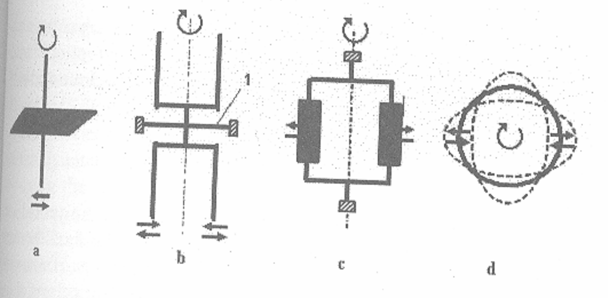

Fig. 1. Resonating structures of the MEMS-based gyros: a) "cloverleaf" resonator; b) double-ended tuning fork; c) vibrating plate; d) vibrating ring

As relative displacements of the tuning fork with respect to the rigid rotating frame are being considered, the full acceleration $\{\mathbf{a}_F\}=\{\mathbf{a}\}+\{\mathbf{a}_N\}+\{\mathbf{a}_T\}+\{\mathbf{a}_C\}$ is used in the virtual work equation of each finite element where $\{\mathbf{a}\}$ is relative acceleration with respect to the rotating frame; $\{\mathbf{a}_N\}$, $\{\mathbf{a}_T\}$ are normal and tangential accelerations due to the rotation of the frame; $\{\mathbf{a}_C\}$ is Coriolis acceleration. The structural dynamic equation of the finite element of a MEMS-based gyros is obtained as [7, 8]:

$$[\mathbf{M}]\{\ddot{\mathbf{U}}\}+(2\Omega[\mathbf{G}]+[\mathbf{C}])[\{\dot{\mathbf{U}}\}+\left([\mathbf{K}]-\Omega^2[\mathbf{K}_1]+\varepsilon[\mathbf{G}]\right)\{\mathbf{U}\}=\{\mathbf{R}\}+\{\mathbf{F}\}+\Omega^2[\mathbf{K}_1]\{\mathbf{X}\}-\varepsilon[\mathbf{G}]\{\mathbf{X}\}; \qquad (2)$$

Where $[\mathbf{G}]$ is gyroscopic matrix, $[\mathbf{K}_1]$ is centripetal force matrix, $[\mathbf{M}]$, $[\mathbf{K}]$ are correspondingly mass and stiffness matrices, $\{\mathbf{F}\}$ is nodal excitation forces due to the inverse piezoelectric effect, $\{\mathbf{X}\}$ is vector of nodal coordinates of the finite element, Ω and ε correspondingly are the angular velocity and angular acceleration of the outer (reference) frame rotation about its Oz axis. The structural damping forces are assumed to be very small and are expressed by means of the proportional damping matrix $[\mathbf{C}]=\alpha[\mathbf{M}]+\beta[\mathbf{K}]$.

By the use of finite element analysis both the operation specifics of the sensor and the quantitative evaluation of the relationship of the output signal against the angular velocity of the outer frame have been carried out.

By using the finite element model of the tuning fork–shaped quartz transducer we calculated a finite number of modes of a non-rotating transducer. They are obtained by solving the characteristic equation as:

$$\left([\mathbf{K}]-\omega^2[\mathbf{M}]\right)\{\mathbf{U}\}=0\,. \qquad (3)$$

Parametric Sensitivity

We consider a computational model of the sensor that uses a micromachined quartz element - a vibrating quartz tuning fork - to measure angular rotational velocity (Fig. 1b). Its performance and dynamic features depend substantially upon modal frequencies and shapes of vibration. The sensitivity is defined in terms of a sensitivity function, which denotes the sensitivity of the output values of the system to the variations of the system's design parameters. The finite element matrices of the transducer can be presented as functions $[\mathbf{K}(b_i)],[\mathbf{M}(b_i)]$ of the design parameters b_i of the vibrating structure. By using the techniques of optimum control and design, linear relations between small variations of the design parameters and corresponding variations of modal frequencies are obtained [8].

The sensitivity functions obtained can be further employed to the shape optimization of the tuning fork in order to satisfy the necessary ratio between the working resonant frequencies as well as to ensure the necessary vibration shapes and to determine which modifications would be the most effective for the desired change.

It is evident from the study performed that the sensor appears to be sensitive to design parameter deviations. For a tuning-fork shaped MEMS-based gyro presented in Fig.1b the analysis of the sensitivity coefficients indicates, in particular, that the stiffness of the supporting bar 1 has the main influence on the 3rd and 4th modal frequencies. This circumstance requires special attention during the dynamic analysis, as here the magnitudes of the two modal frequencies are very close and may interchange as a result of a small variation of design parameters.

Analysis of the Dynamic Behavior of Tuning-fork Shaped MEMS-based Gyro

In order to perform its function as the sensitive element of the angular rate meter, the transducer under consideration is excited by means of the applied electric voltage over one half of the fork (input tines). The frequency of excitation of the fork is close to the natural frequencies of resonant out-of-plane modes 3 and 4, though the modes are not excited because of the in-plane action of the electromechanical excitation forces. For in-plane vibration, the excitation frequency is far below the resonance, so they may be regarded as non-resonant.

The optimum separation of modal frequencies f_3 and f_4 by selecting proper geometrical parameters allows one to obtain the out-of-plane vibration of output tines, the AFCH of which has a plateau or a local minimum on its top. Thus the tolerance of the excitation frequency is allowable in a wider range, Fig. 2a, where $\bar{U}_A, \bar{U}_B$ and ϕ_A, ϕ_B denote the amplitudes and phases of input and output tines' vibration.

If f_3 is very close but not equal to f_4, and the out-of-plane vibration of output tines will be excited by the rotation of the frame, the phase of vibration of output tines will depend on the mutual positions of natural frequencies of symmetrical (f_3) and anti-symmetrical (f_4) out-of-plane modes. As the frequency values of them interchange (i.e., f_3 becomes greater than f_4), the phase angle of output tines vibration changes through value π, Fig. 2b. This leads to the same effect as the change of the sign of the angular velocity of the frame rotation and may cause misinterpretations of the direction of rotation. By ensuring an appropriate separation between modal frequencies of the out-of-plane vibration of the transducer, the peaks of the AFCH may be made wider and less sharp in order to decrease the possibility of misinterpretation of the measured angular velocity direction that is identified on the base of the phase angle of vibration of the output tines.

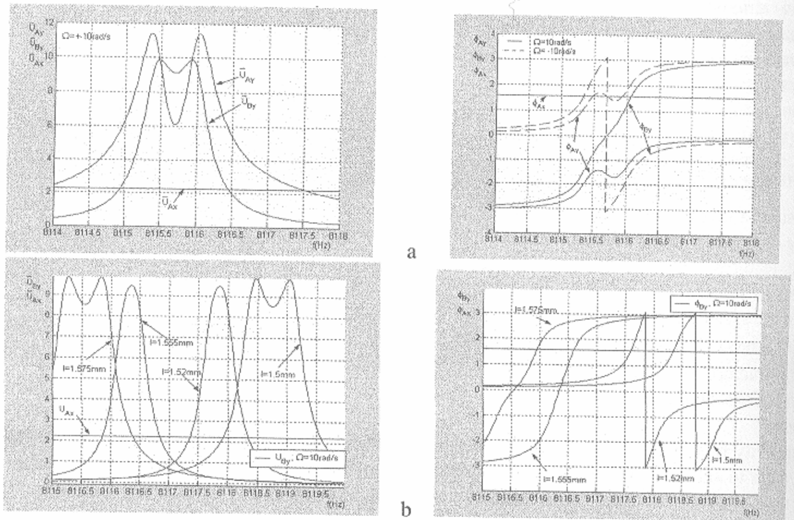

Fig.2. Effects of separation of modal frequencies f_3 and f_4 on the frequency response of the transducer: a) f_3.=8115,3 Hz; f_4.=8116 Hz; b) frequencies f_3 and f_4 obtained at various values of the length of the vibrating tine, l.

Conclusions

Structural vibration problems present a major hazard and design limitation for interpretation of both the MEMS-based system properties and their impact on system performance.

A novel computation model and software tools have been developed and applied to the analysis of the dynamic behavior of the balanced H-shape oscillator of the piezoelectric angular rate sensor by using the finite element method and including the gyroscopic effects upon vibrating structure in a rotating frame. The model is capable to predict the system behavior and allows for a comprehensive characterization of the system, with respect to both static and dynamic imperfections.

It has been demonstrated that investigation of the dynamic behavior of an angular rate sensor by employing the 3D FE model facilitates considerably the understanding of the operation specifics of the sensor and allows for the quantitative evaluation of the input-output relationship. Main operation modes of the angular rate sensor have been analyzed and optimum separation of the modal frequency values have been found ensuring stable operation law, as well as, correct interpretation of the direction of the measured angular velocity.

The modeling approach worked out can also be applied to the education of advanced level specialists in the field of measurement and instrumentation in the design-oriented framework.

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